Functions of random variables, Expectation

Putting a value on random variables

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Review: functions of random variables

Review: Expectation

Definition. *The expectation of a numeric random variable* $X = (\Omega, \mathbb{P})$ *is given by the following:*

$$\mathrm{E}[X] := \sum_{x \in \Omega} x \mathbb{P}(X = x).$$

Expectation of (functions of) random variables

Linearity of expectation

Theorem. *Let* X, Y *be two (numeric) random variables, and their joint distribution given by* $\mathbb{P}(X = x, Y = y)$. *The expectation is a "linear operator": that is,*

1. $\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y]$,
2. $\mathrm{E}[cX] = c\mathrm{E}[X]$ for any fixed $c \in \mathbb{R}$.

Some important distributions

Review: Bernoulli

Review: Binomial

Geometric

Expectation equality

Let X be a random variable. If its sample space $\Omega \subset \mathbb{N}$, the following equality holds:

$$\mathrm{E}[X] = \sum_{i=0}^{\infty} \mathbb{P}(X \geq i).$$

Expectation equality

Geometric revisited

Poisson

Poisson

Sum of Poisson

Poisson as limit of binomial