CS 70 Discrete Mathematics and Probability Theory
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DIS 1A

1 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

2 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

3 Necessary and Sufficient Conditions

(a) Given implication $A \implies B$, A is a _____ condition for B.

(b) Given implication $\neg A \implies \neg B$, A is a _____ condition for B.

(c) Given implication $\neg B \implies \neg A$, A is a _____ condition for B.

(d) Given implication $B \implies A$, A is a _____ condition for B.

4 Logical Equivalence?

Decide whether each of the following logical equivalences is correct and justify your answer.

(a) $\forall x \left(P(x) \wedge Q(x)\right) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$

(b) $\forall x \left(P(x) \vee Q(x)\right) \equiv \forall x\, P(x) \vee \forall x\, Q(x)$

(c) $\exists x \left(P(x) \vee Q(x)\right) \equiv \exists x\, P(x) \vee \exists x\, Q(x)$

(d) $\exists x \left(P(x) \wedge Q(x)\right) \equiv \exists x\, P(x) \wedge \exists x\, Q(x)$