

1 Short Answers

- (a) A connected planar simple graph has 5 more edges than it has vertices. How many faces does it have?
- (b) How many edges need to be removed from a 3-dimensional hypercube to get a tree?

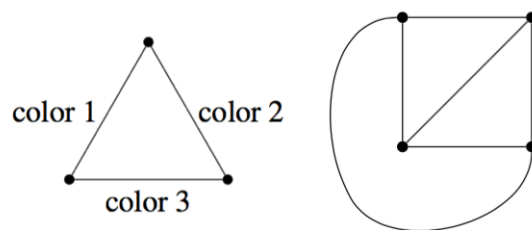
2 Hamiltonian Tour in a Hypercube

An alternative type of tour to an Eulerian Tour in graph is a Hamiltonian Tour: a tour that visits every vertex exactly once. Prove or disprove that the hypercube contains a Hamiltonian cycle, for hypercubes of dimension $n \geq 2$.

Hint: When proceeding by induction, a good place to start is writing out what this tour would look like in a 3-dimensional hypercube when starting from the 000 vertex, and using the recursive definition of an n -dimensional hypercube.

3 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.



- (a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers 1, 2, 3 for colors. A figure is shown on the right.)
- (b) Prove that any graph with maximum degree $d \geq 1$ can be edge colored with $2d - 1$ colors.
- (c) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.