

1 Probability Potpourri

Provide brief justification for each part.

(a) For two events A and B in any probability space, show that $\mathbb{P}[A \setminus B] \geq \mathbb{P}[A] - \mathbb{P}[B]$.

(b) Suppose $\mathbb{P}[D | C] = \mathbb{P}[D | \bar{C}]$, where \bar{C} is the complement of C . Prove that D is independent of C .

(c) If A and B are disjoint, does that imply they're independent?

3 Balls and Bins

Throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Let A be the event that at least k bins are empty. Notice that there are m subsets of k bins out of the total n bins. If we assume A_i is the event that the i^{th} set of k bins is empty. Then we can write A as the union of A_i 's.

$$A = \bigcup_{i=1}^m A_i.$$

Use the union bound to give an upper bound on the probability A from part (c).

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?