

1 Diversify Your Hand

You are dealt 5 cards from a standard 52 card deck. Let X be the number of distinct values in your hand. For instance, the hand (A, A, A, 2, 3) has 3 distinct values.

- (a) Calculate $\mathbb{E}[X]$.
- (b) Calculate $\text{Var}(X)$.

2 Covariance

- (a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let X_1 and X_2 be indicator random variables for the events of the first and second ball being red, respectively. What is $\text{cov}(X_1, X_2)$? Recall that $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

- (b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let X_1 and X_2 be indicator random variables for the events of the first and second draws being red, respectively. What is $\text{cov}(X_1, X_2)$?

3 Correlation and Independence

- (a) What does it mean for two random variables to be uncorrelated?
- (b) What does it mean for two random variables to be independent?
- (c) Are all uncorrelated variables independent? Are all independent variables uncorrelated? If your answer is yes, justify your answer; if your answer is no, give a counterexample.