

1 Continuous Computations

Let X be a continuous random variable whose PDF is cx^3 (for some constant c) in the range $0 \leq x \leq 1$, and is 0 outside this range.

(a) Find c .

(b) Find the CDF of X .

(c) Find $\mathbb{E}(X)$.

2 Max of Uniforms

Let X_1, \dots, X_n be independent $U[0, 1]$ random variables, and let $X = \max(X_1, \dots, X_n)$. Compute each of the following in terms of n .

(a) What is the cdf of X ?

- (b) What is the pdf of X ?
- (c) What is $\mathbb{E}[X]$?
- (d) What is $\text{Var}[X]$?

3 Darts Again

Edward and Khalil are playing darts on a circular dartboard.

Edward's throws are uniformly distributed over the entire dartboard, which has a radius of 10 inches. Khalil has good aim; the distance of his throws from the center of the dartboard follows an exponential distribution with parameter $\frac{1}{2}$.

Say that Edward and Khalil both throw one dart at the dartboard. Let X be the distance of Edward's dart from the center, and Y be the distance of Khalil's dart from the center of the dartboard. What is $\mathbb{P}[X < Y]$, the probability that Edward's throw is closer to the center of the board than Khalil's? Leave your answer in terms of an unevaluated integral.

[*Hint:* X is not uniform over $[0, 10]$. Solve for the distribution of X by first computing the CDF of X , $\mathbb{P}[X < x]$.]