1 Condition on an Event

The random variable X has the PDF

$$f_X(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the value of c.

(b) Let A be the event $\{X > 1.5\}$. Calculate $\mathbb{P}(A)$ and the conditional PDF of X given that A has occurred.

2 Joint Practice

Suppose that X and Y are random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} Ax^2y^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where A is a positive constant.

(a) What is the value of A?

(b) What is the marginal density of X?

(c) What is $\text{cov}(X,Y)$?

3 Darts with Friends

Michelle and Alex are playing darts. Being the better player, Michelle's aim follows a uniform distribution over a disk of radius 1 around the center. Alex's aim follows a uniform distribution over a disk of radius 2 around the center.

(a) Let the distance of Michelle's throw from the center be denoted by the random variable X and let the distance of Alex's throw from the center be denoted by the random variable Y.

- What's the cumulative distribution function of X?
- What's the cumulative distribution function of Y?
- What's the probability density function of X?
- What's the probability density function of Y?

(b) What's the probability that Michelle's throw is closer to the center than Alex's throw? What's the probability that Alex's throw is closer to the center?

(c) What's the cumulative distribution function of $U = \max\{X,Y\}$?

(d) What's the cumulative distribution function of $V = \min\{X,Y\}$?